



Math Virtual Learning

Probability and Statistics

May 13, 2020



Probability and Statistics

Lesson: May 13, 2020

Objective/Learning Target:

Students will be able to distinguish if two occurring events are Independent or Dependent

Let's Get Started!

The letters from a Scrabble game were placed upside down next to the board. In a standard Scrabble game, there are 100 tiles. There are 2 blank tiles and letter tiles. Below is the number of tiles for of each letter include in the game.

A = 9	N = 6
B = 2	O = 8
C = 2	P = 2
D = 4	Q = 1
E = 12	R = 6
F = 2	S = 4
G = 3	T = 6
H = 2	U = 4
I = 9	V = 2
J = 1	W = 2
K = 1	X = 1
L = 4	Y = 2
M = 2	Z = 1

What is the probability of getting the following tiles if you draw 1st?

$P(A)$

$P(R)$

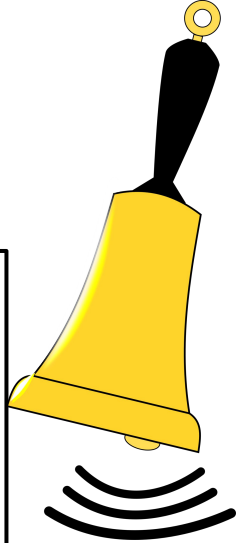
$P(B \text{ or } C)$

$P(\text{Vowel})$

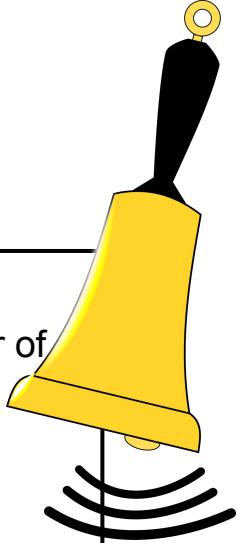
$P(\text{Blank})$

$P(\text{NOT } W)$

Why will the probability change if you have to draw second?



Let's Get Started **ANSWERS!**



The letters from a Scrabble game were placed upside down next to the board. In a standard Scrabble game, there are 100 tiles. There are 2 blank tiles and letter tiles. Below is the number of tiles for of each letter include in the game.

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I = 9	V = 2
J = 1	W = 2
K = 1	X = 1
L = 4	Y = 2
M = 2	Z = 1

What is the probability of getting the following tiles if you draw 1st?

$$P(A) = 9/100$$

$$P(R) = 6/100 = 3/50$$

$$P(B \text{ or } C) = 4/100 = 1/25$$

$$P(\text{Vowel } a/e/i/o/u) = 42/100 = 21/50$$

$$P(\text{Blank}) = 2/100 = 1/50$$

$$P(\text{NOT } W) = 98/100 = 49/50$$

Why will the probability change if you have to draw second?

Once the first person gets to go, there will no longer be 100 tiles in the pile and they may have taken some of the letters listed above.

Independent vs Dependent Probability

Today we are going to take a look at two events to determine if they are Independent or Dependent Probabilities.

Before we start let's look at a couple of vocabulary terms you will need to be familiar with:

Event: The outcome from one experiment.

Independent: The first event doesn't affect the second event. (or the 1st outcome doesn't affect the 2nd outcome)

Dependent: the first event does affect the second event. (the 1st outcome does affect the 2nd outcome) What happens the 2nd time depends on what happened the 1st time.

Replacement: After you do an experiment and record the events, you put everything back to its original state before conducting the 2nd experiment.

Example 1:

Example: Two number cubes (a red one and a blue one) are rolled. What is the probability that the outcome of the blue number cube is a 5 and the red number cube is even?

P(blue number cube 5):

P(red number cube even):



Would these be Independent or Dependent Events?

These events would be **INDEPENDENT** because they are two separate cubes. What you roll on the blue cube will no way impact how the red cube rolls.

Example 2:

Example: There are 6 black socks and 4 white socks in a drawer. If one sock is chosen without looking and thrown on the dresser and then a second one is taken out, what is the probability that both will be black?

P(1st sock black)

P(2nd sock black)



Would these be Independent or Dependent events?

These would be **DEPENDENT** events. What you draw on the first pick will impact your chances on the second pick. If you draw a black out the first time, then there are less black socks in the drawer for the second pick. If you pick a white sock out the first time, there are just as many black socks the second time.

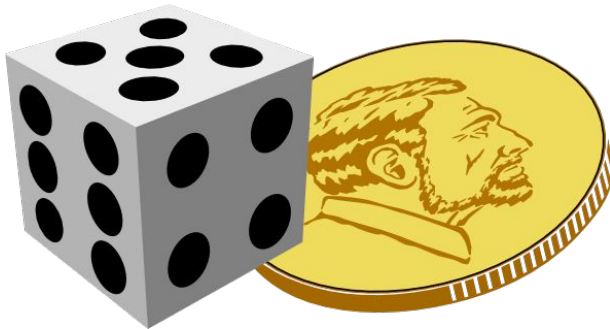
The first event will make a difference for the second event.

Example 3:

You are playing a game with your sister where you flip a coin and roll a die. You win if the coin lands on heads AND the die rolls an even number.

Would these be Independent or Dependent events?

These would be **INDEPENDENT**. Because there are two separate items, the result of the coin will not impact the result of the die.



Example 4:

Your school is holding a raffle at the football game with two prizes. After the first ticket is pulled out and the winner is determined, the ticket is taped to the prize. The next ticket is pulled out to determine the winner of the second prize.



Would these be Independent or Dependent events?

These would be **DEPENDENT** events. What you draw on the first pick will impact your chances on the second pick. After the first ticket is drawn, since it is not put back, there is one less ticket to pick from the 2nd time. If I only bought 1 ticket, and mine is drawn the first time, then I don't have any chances of winning the second prize..

The first event will make a difference for the second event.

Example 5:

Your teacher has a deck of cards and will be drawing one at random. If the suit of the card she draws matches the card you have, then you will have to answer her question. Each time she draws a card, she puts it back in the deck and shuffles. What is the probability that you will be called on twice in a row?



Would these be Independent or Dependent events?

These would be **INDEPENDENT** because your teacher put the card back in the deck. So each time your teacher draws a new card, there is a full deck. What your teacher drew last time will have no impact on what your teacher will draw this time.

YOUR TURN >>>>>>

For each of the practice problems on the following pages, determine if the series of events is Independent or Dependent.

Then answer the probability questions



Practice 1:

A standard coin is tossed three times. You would like to get a heads at least two of those times.

- Would these be Independent or Dependent Events?
- What is the probability of getting a heads the first flip?
- If you got a heads on the first and second flip, what is the probability of getting a heads on the third flip?



Practice 1 ANSWER:

A standard coin is tossed three times. You would like to get a heads at least two of those times.



- Would these be Independent or Dependent Events? **INDEPENDENT** - the coin is flipped fresh each time. What it landed on the first time will have no impact on future flips
- What is the probability of getting a heads the first flip? **$1/2$**
- If you got a heads on the first and second flip, what is the probability of getting a heads on the third flip? **$1/2$** - because this is **INDEPENDENT**, it doesn't matter what happened the first two times, there is still a 50-50 ($1/2$) chance of getting heads

Practice 2:

Two cards are drawn from the top of a well-shuffled deck, without putting the first one back.

- Would these be Independent or Dependent Events?
- What is the probability of getting a black Ace on the first draw?
- If you did get a black Ace on the first draw, what is the probability of also getting a black Ace on the second draw?



Practice 2 ANSWER:

Two cards are drawn from the top of a well-shuffled deck, without putting the first one back.

- Would these be Independent or Dependent Events? **DEPENDENT**- If you don't put the first card back, there is one less card in the deck to choose from the 2nd time so it will affect your 2nd draw.
- What is the probability of getting a black Ace on the first draw?
2 Black Aces in a deck of cards (Ace of Clubs & Ace of Spades) & 52 cards in a deck (minus Jokers). So Probability of Black Ace = $2/52$
- If you did get a black Ace on the first draw, what is the probability of also getting a black Ace on the second draw?
If you choose a black ace then there is one less black ACE in the deck the 2nd time & 1 less card in the deck so $1/51$



Practice 3:

A magician is selecting audience members from the crowd to participate in his act. He is going to select one from the left side of the audience, which has 34 people, and one from the right side of the audience, which has 41 people. You are sitting on the right side and DON'T want to be chosen. So you do a quick calculation of the chances you will be picked.

- Would these be Independent or Dependent Events?
- What is the probability of someone on the left side being chosen?
- If a female was picked from the left side, what are the chances you will be picked?



Practice 3 ANSWER:



A magician is selecting audience members from the crowd to participate in his act. He is going to select one from the left side of the audience, which has 34 people, and one from the right side of the audience, which has 41 people. You are sitting on the right side and DON'T want to be chosen. So you do a quick calculation of the chances you will be picked.

- Would these be Independent or Dependent Events? **INDEPENDENT** - the two sides of the audience are separate and so one pick won't affect the other pick.
- What is the probability of someone on the left side being chosen? **There are 34 people on the left side so the probability of one of them being chosen is $\frac{1}{34}$**
- If a female was picked from the left side, what are the chances you will be picked? **$\frac{1}{41}$ - Because these events were INDEPENDENT, it didn't matter who was picked on the left side, you will have a $\frac{1}{41}$ chance of being chosen because there are 41 people on your side of the audience.**

Practice 4:

A bag contains 9 red balls, 5 green balls, and 6 blue balls.

We will be drawing two balls without putting the first one back in the bag.

- Would these be Independent or Dependent Events?
- What is the probability of drawing a green ball the first time?
- If a green ball was drawn, what are the chances a red one will be drawn the second time?



Practice 4 ANSWER:

A bag contains 9 red balls, 5 green balls, and 6 blue balls.

We will be drawing two balls without putting the first one back in the bag.

- Would these be Independent or Dependent Events? **DEPENDENT** because once you draw a marble out and NOT put it back, the chances for the next draw is affected because the total number of balls will be different
- What is the probability of drawing a green ball the first time? There are 20 total marbles in the bag and 5 of them are green so the probability would be $5/20$ or $1/4$
- If a green ball was drawn, what are the chances a red one will be drawn the second time? $6/19$ - Because you didn't put the ball back in, there are longer 20 balls in the bag there are only 19.



Practice 5:

A local charity is hosting a fundraiser. When you purchased your entry ticket it was printed on red paper with the number 10 in the corner. Upon arrival you realized that they are giving away a prize. They will spin the spinner below and then draw a number between 1 and 30 out of a hat. If the color and number match your ticket, you win a cruise.



- Would these be Independent or Dependent Events?
- What is the probability of your color being chosen?
- If your color is chosen, what is the probability of your number being chosen?

Practice 5 ANSWER:



A local charity is hosting a fundraiser. When you purchased your entry ticket it was printed on red paper with the number 10 in the corner. Upon arrival you realized that they are giving away a prize. They will spin the spinner below and then draw a number between 1 and 30. If the color and number match yours, you win a cruise.

- Would these be Independent or Dependent Events? **INDEPENDENT** - because the spinner and the number draw are separate.
- What is the probability of your color being chosen? **$1/6$** - there are 6 colors
- If your color is chosen, what is the probability of your number being chosen? **$1/30$** - because these are **INDEPENDENT** events, it really doesn't matter what color was spun...there is still a $1/30$ chance of your number being drawn.

Practice 6:

In a class of 7, there are 4 students who play soccer. If the teacher chooses 3 students, what is the probability that none of the three of them play soccer?

- Would these be Independent or Dependent Events?
- What is the probability that the first student chosen does not play soccer?
- If you did choose a student who does not play soccer, what is the probability that the 2nd student chosen also does not play soccer?





Practice 6 ANSWER:

In a class of 7, there are 4 students who play soccer. If the teacher chooses 3 students, what is the probability that none of the three of them play soccer?

- Would these be Independent or Dependent Events? **DEPENDENT**- once a student is chosen they cannot be chosen again.
- What is the probability that the first student chosen does not play soccer? Out of the 7 students, 4 play soccer so three do not. The probability of not playing soccer would be $3/7$
- If you did choose a student who does not play soccer, what is the probability that the 2nd student chosen also does not play soccer? Since you have already chosen 1 student who does not play soccer, now you have 1 less to choose from so probability that the 2nd student does not play soccer = $2/6$

Practice 7:

The letters to the word PROBABILITY were cut up and put into a bag. Three of the letters will be drawn at random and not replaced into the bag.

- Would these be Independent or Dependent Events?
- What is the probability of the letter B being chosen the first time?
- If the letters B and L were chosen the first time, what is the probability that the letter B will be chosen the third time?



Practice 7 ANSWERS:



The letters to the word PROBABILITY were cut up and put into a bag. Three of the letters will be drawn at random (one at a time) and not replaced into the bag.

- Would these be Independent or Dependent Events? **DEPENDENT** because you are drawing from the same bag (not separate) and not replacing the piece, the second time you draw the possibilities will be different.
- What is the probability of the letter B being chosen the first time? **There are 11 letters in probability and 2 of them are a B so the probability is $\frac{2}{11}$**
- If the letters B and L were chosen the first & second time, what is the probability that the letter B will be chosen the third time? **$\frac{1}{9}$ - out of the 11 letters, two have already been drawn and not put back so there are only 9 left in the bag. One of those two was a B so there is only one B left in the bag.**

Practice 8:

Billy is playing in his sandbox. He enjoys burying his toys in the sand. He has 3 dump trucks, 2 tractors, and 2 pickup trucks. Billy randomly selects a toy, buries it, then chooses another. What is the probability that the first toy he picks is a tractor and so is the second?

- Would these be Independent or Dependent Events?
- What is the probability that Billy chooses a tractor the first time?
- If he chooses a tractor the first time, what is the probability that he randomly chooses a tractor again the 2nd time?



Practice 8 ANSWER:



Billy is playing in his sandbox. He enjoys burying his toys in the sand. He has 3 dump trucks, 2 tractors, and 2 pickup trucks. Billy randomly selects a toy, buries it, then chooses another. What is the probability that the first toy he picks is a tractor and so is the second?

- Would these be Independent or Dependent Events? **DEPENDENT** - if Billy buries the first one, then there is one less the 2nd time he picks up a toy.
- What is the probability that Billy chooses a tractor the first time? **7 total toys and two of them are tractors so the probability of choosing a tractor the first time = $2/7$**
- If he chooses a tractor the first time, what is the probability that he randomly chooses a tractor again the 2nd time? **Since 1 tractor is already buried it would be $1/6$**

Practice 9:

A die is rolled and a card is drawn from a standard 52 deck of cards.

- Would these be Independent or Dependent Events?
- What is the probability of an even number being rolled?
- If an odd number was rolled, what is the probability of the Three of Hearts being drawn?



Practice 9 ANSWER:



A die is rolled and a card is drawn from a standard 52 deck of cards.

- Would these be Independent or Dependent Events? **INDEPENDENT** - because these are two separate objects, there events will not be connected
- What is the probability of an even number being rolled? Since there are three even numbers on a die the probability would be $3/6 = 1/2$
- If an odd number was rolled, what is the probability of the Three of Hearts being drawn? $1/52$ - because these events are **INDEPENDENT**, it doesn't matter what was rolled. There is only one 3 of hearts in the deck so the probability is $1/52$.

Practice 10:

You are playing twister and you are hoping that anything is spun EXCEPT a left foot red spot.



- Would these be Independent or Dependent Events?
- What is the probability of your left foot being spun?
- If the spinner does land in the left foot area, what is the probability that the red will be spun?

Practice 10 ANSWERS:

You are playing twister and you are hoping that anything is spun EXCEPT a left foot red spot.



- Would these be Independent or Dependent Events? This is a tricky one. Because there is a color in each of the four sections, the chances of getting a red is the same no matter which hand/foot is spun. So these are **INDEPENDENT**.
- What is the probability of your left foot being spun? Because there are four hand/foot areas, the probability is $1/4$
- If the spinner does land in the left foot area, what is the probability that the red will be spun? $1/4$ - in the left foot area, there are four colors and one of them is a red.